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In ΔABC the following relationship holds:

$$\frac{\sin A\sin B}{\sin^2\frac{C}{2}}+\frac{\sin B\sin C}{\sin^2\frac{A}{2}}+\frac{\sin C\sin A}{\sin^2\frac{B}{2}}\geq 9$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\frac{\sin A\sin B}{\sin^2\frac{C}{2}}+\frac{\sin B\sin C}{\sin^2\frac{A}{2}}+\frac{\sin C\sin A}{\sin^2\frac{B}{2}}=$$

$$=\frac{1}{4R^2}\sum\frac{a^2b^2}{(s-a)(s-b)}=\frac{1}{4R^2.sr^2}\sum a^2b^2(s-c)$$

$$=\frac{1}{4R^2r^2s}\left[s\sum a^2b^2-abc\sum ab\right]\overset{\sum x^2\geq\sum xy}{\geq}$$

$$\geq\frac{1}{4R^2r^2s}\left[s.abc\sum a-abc\sum ab\right]\geq\frac{1}{4R^2r^2s}abc[2s^2-s^2-r^2-4Rr]\overset{Gerretsen}{\geq}$$

$$\geq\frac{4Rrs}{4R^2r^2s}[12Rr-6r^2]=12-\frac{6r}{R}\geq 12-6.\frac{1}{2}(Euler)=9$$

Equality holds for $a=b=c$.