ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$m_a sinA + m_b sinB + m_c sinC \leq \frac{9\sqrt{3}R}{4}$$

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WLOG: $a \leq b \leq c$

$$a \leq b \leq c \Rightarrow A \leq B \leq C \Rightarrow sinA \leq sinB \leq sinC$$

$$a \leq b \leq c \Rightarrow m_a \geq m_b \geq m_c$$

$$\sum_{cyc} m_a sinA \leq \frac{1}{3} \cdot \sum_{cyc} m_a \cdot \sum_{cyc} sinA = \frac{1}{3} \cdot \frac{s}{R} \cdot \sum_{cyc} m_a \leq$$

$$\overset{GOTMAN}{\leq} \frac{s}{3R} \cdot \frac{9R}{2} = \frac{3s}{2} \overset{MITRINOVIC}{\leq} \frac{3}{2} \cdot \frac{3\sqrt{3}}{2} \cdot R = \frac{9\sqrt{3}R}{4}$$

Equality holds for $a = b = c$.