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In ΔABC the following relationship holds:

$$\left(1+9\tan^2\frac{A}{2}\right)\left(1+9\tan^2\frac{B}{2}\right)\left(1+9\tan^2\frac{C}{2}\right)\geq 64$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\left(1+9\tan^2\frac{A}{2}\right)\left(1+9\tan^2\frac{B}{2}\right)\left(1+9\tan^2\frac{C}{2}\right)\geq 64$$

$$1+9\sum\tan^2\frac{A}{2}+81\sum\tan^2\frac{A}{2}\tan^2\frac{B}{2}+729\prod\tan^2\frac{A}{2}\geq 64$$

$$1+9\left[\left(\frac{4R+r}{s}\right)^2-2\right]+81\left[\frac{s^2-2r^2-8Rr}{s^2}\right]+729\frac{r^2}{s^2}\geq 64$$

$$9(4R+r)^2-648Rr-162r^2+729r^2\geq 0$$

$$(4R+r)^2-72Rr+63r^2\geq 0$$

$$\left(\frac{R}{r}-2\right)^2\geq 0$$

$$Equality\ \ holds\ for\ A=B=C=\frac{\pi}{3}$$