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In ΔABC the following relationship holds:

$$r_a\left(\sin\frac{B}{2}+\sin\frac{C}{2}\right)+r_b\left(\sin\frac{C}{2}+\sin\frac{A}{2}\right)+r_c\left(\sin\frac{A}{2}+\sin\frac{B}{2}\right)\le\frac{9R}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum\sin\frac{A}{2}\overset{Jensen}{\le}3\sin\left(\frac{A+B+C}{6}\right)=\frac{3}{2},$$

$$WLOG\ a\ge b\ge c\ then\ r_a\ge r_b\ge r_c\ and\ \sin\frac{A}{2}\ge\sin\frac{B}{2}\ge\sin\frac{C}{2}$$

$$r_a\left(\sin\frac{B}{2}+\sin\frac{C}{2}\right)+r_b\left(\sin\frac{C}{2}+\sin\frac{A}{2}\right)+r_c\left(\sin\frac{A}{2}+\sin\frac{B}{2}\right)\overset{Chebyshev}{\le}$$

$$\le\frac{1}{3}\left(\sum r_a\right)\left(\sum\left(\sin\frac{B}{2}+\sin\frac{C}{2}\right)\right)\le\frac{1}{3}(4R+r).2.\frac{3}{2}\overset{Euler}{\le}\frac{9R}{2}$$

Equality holds for $a=b=c$.