ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a}{m_b+m_c}\cot\frac{A}{2}+\frac{m_b}{m_c+m_a}\cot\frac{B}{2}+\frac{m_c}{m_a+m_b}\cot\frac{C}{2}\ge\frac{3\sqrt{3}}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$WLOG\ a\ge b\ge c$$

$$then\ m_a\le m_b\le m_c,\ m_a+m_b\le m_a+m_c\le m_b+m_c,$$
$$\cot\frac{A}{2}\le\cot\frac{B}{2}\le\cot\frac{C}{2}$$

$$\frac{m_a}{m_b+m_c}\cot\frac{A}{2}+\frac{m_b}{m_c+m_a}\cot\frac{B}{2}+\frac{m_c}{m_a+m_b}\cot\frac{C}{2}\overset{Chebyshev}{\ge}$$

$$\ge\frac{1}{3}\sum\frac{m_a}{m_b+m_c}\sum\cot\frac{A}{2}\overset{Nesbitt}{\ge}$$

$$\ge\frac{1}{3}\cdot\frac{3}{2}\cdot\frac{s}{r}\overset{Mitrinovic}{\ge}\frac{1}{2}\cdot\frac{3\sqrt{3}r}{r}=\frac{3\sqrt{3}}{2}$$

$$Equality\ for\Delta ABC\ equilateral$$