ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$3(ab+bc+ca)\geq a^2+b^2+c^2+36Rr$$

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$$ab+bc+ca=s^2+r^2+4Rr$$

$$a^2+b^2+c^2=2(s^2-r^2-4Rr)$$

We must prove that:

$$3(s^2+r^2+4Rr)\geq 2(s^2-r^2-4Rr)+36Rr$$

$$3s^2+3r^2+12Rr\geq 2s^2-2r^2-8Rr+36Rr$$

$$s^2\geq 16Rr-5r^2$$

which is Gerretsen's inequality.

Equality holds for $a=b=c$.