ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{h_b+h_c}{a}+\frac{h_c+h_a}{b}+\frac{h_a+h_b}{c}\le 3\sqrt{3}$$

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Solution by Daniel Sitaru-Romania

$$\frac{h_b+h_c}{a}+\frac{h_c+h_a}{b}+\frac{h_a+h_b}{c}=\sum_{cyc}\frac{h_b+h_c}{a}=$$

$$=\sum_{cyc}\frac{\frac{2F}{b}+\frac{2F}{c}}{a}=2F\sum_{cyc}\frac{\frac{1}{b}+\frac{1}{c}}{a}=2F\sum_{cyc}\frac{b+c}{abc}=$$

$$=\frac{2F}{abc}\sum_{cyc}(b+c)=\frac{2F}{4RF}\cdot 2\sum_{cyc}a=\frac{1}{R}\cdot\sum_{cyc}a=$$

$$=\frac{1}{R}\cdot 2s\overset{MITRINOVIC}{\overbrace{\le}}\frac{1}{R}\cdot 2\cdot\frac{3\sqrt{3}}{2}\cdot R=3\sqrt{3}$$

Equality holds for $a=b=c$.