ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{h_a+h_b}{a+b}+\frac{h_b+h_c}{b+c}+\frac{h_c+h_a}{c+a}\le\frac{3\sqrt{3}}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{h_a+h_b}{a+b}+\frac{h_b+h_c}{b+c}+\frac{h_c+h_a}{c+a}=\sum_{cyc}\frac{h_a+h_b}{a+b}=$$

$$=\sum_{cyc}\frac{\frac{2F}{a}+\frac{2F}{b}}{a+b}=2F\sum_{cyc}\frac{\frac{1}{a}+\frac{1}{b}}{a+b}=2F\sum_{cyc}\frac{1}{ab}=$$

$$=2F\cdot\frac{a+b+c}{abc}=2F\cdot\frac{2s}{4RF}=\frac{s}{R}\overbrace{\le}^{MITRINOVIC}\frac{3\sqrt{3}R}{2}\cdot\frac{1}{R}=\frac{3\sqrt{3}}{2}$$

Equality holds for $a=b=c$.