ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum a\left(\sin\frac{B}{2}+\sin\frac{C}{2}\right) \leq 3\sqrt{3}R$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$WLOG\ a \geq b \geq c\ then\ \sin\frac{A}{2} \geq \sin\frac{B}{2} \geq \sin\frac{C}{2}\ and$$

$$\sin\frac{A}{2}+\sin\frac{B}{2} \geq \sin\frac{B}{2}+\sin\frac{C}{2} \geq \sin\frac{C}{2}+\sin\frac{A}{2}$$

$$\sum a\left(\sin\frac{B}{2}+\sin\frac{C}{2}\right) \overset{Chebyshev}{\leq} \frac{1}{3}\left(\sum a\right)\left(\sum \sin\frac{A}{2}+\sin\frac{B}{2}\right) = \frac{2s}{3}.2\left(\sum \sin\frac{A}{2}\right) \overset{Jensen}{\leq}$$

$$\leq \frac{4s}{3}3\sin\frac{\pi}{6} = 2s \leq 3\sqrt{3}R(\ Mitrinovic)$$

$$Equality\ for\ a = b = c$$