ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a^{m_a}b^{m_b}c^{m_c} \le \left(R\sqrt{3}\right)^{\frac{9R}{2}}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$WLOG\ a \ge b \ge c\ then\ m_a \le m_b \le m_c$$

$$am_a + bm_b + cm_c \overset{Chebyshev}{\le} \frac{1}{3}(a+b+c)(m_a+m_b+m_c) =$$
$$= \frac{1}{3}2s(m_a+m_b+m_c)(1)$$

$$(m_a+m_b+m_c) \overset{Leuenberger}{\le} 4R + r \overset{Euler}{\le} \frac{9R}{2}\ (2)$$

$$\frac{am_a+bm_b+cm_c}{(m_a+m_b+m_c)} \overset{(1)}{\le} \frac{\frac{1}{3}2s(m_a+m_b+m_c)}{(m_a+m_b+m_c)} = \frac{1}{3}2s \overset{Mitrinovic}{\le} \frac{2}{3}\frac{3\sqrt{3}R}{2} = \sqrt{3}R\ (3)$$

$$a^{m_a}b^{m_b}c^{m_c} \overset{AM-GM}{\le} \left(\frac{am_a+bm_b+cm_c}{(m_a+m_b+m_c)}\right)^{(m_a+m_b+m_c)} \overset{(2)\&(3)}{\le} \left(R\sqrt{3}\right)^{\frac{9R}{2}}$$

$$Equality\ for\ a = b = c$$