ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$m_a r_a + m_b r_b + m_c r_c \le \frac{27R^2}{4}$$

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Solution by Tapas Das-India

$$WLOG\ a \ge b \ge c\ then\ m_a \le m_b \le m_c\ and\ r_a \ge r_b \ge r_c$$

$$m_a r_a + m_b r_b + m_c r_c \overset{Chebyshev}{\le} \frac{1}{3}\left(\sum m_a\right)\left(\sum r_a\right) \overset{Leuenberger}{\le}$$

$$\le \frac{1}{3}(4R + r)(4R + r) \overset{Euler}{\le} \frac{1}{3}\left(4R + \frac{R}{2}\right)^2 = \frac{27R^2}{4}$$

$$Equality\ for\ a = b = c$$