ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{a}{b+c}\tan\frac{A}{2} \geq \frac{\sqrt{3}}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$WLOG\ a \geq b \geq c\ \ then \tan\frac{A}{2} \geq \tan\frac{B}{2} \geq \tan\frac{C}{2}\ \ and \frac{a}{b+c} \geq \frac{b}{c+a} \geq \frac{c}{a+b}$$

$$\sum \frac{a}{b+c}\tan\frac{A}{2} \overset{Chebycv}{\geq} \frac{1}{3}\left(\sum \frac{a}{b+c}\right)\left(\sum \tan\frac{A}{2}\right) \overset{Nesbitt}{\geq}$$

$$\geq \frac{1}{3}\cdot\frac{3}{2}\cdot\frac{4R+r}{s} \overset{Doucet}{\geq} \frac{\sqrt{3}}{2}$$

$$Equality\ for\ a = b = c$$