ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum m_a \csc A \geq 6\sqrt{3}r$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum \frac{1}{h_a} = \frac{1}{r}, \sum h_a \overset{AM-HM}{\geq} \frac{9}{:\sum \frac{1}{h_a}} = 9r \quad (1)$$

$$WLOG\ a \geq b \geq c\ then\ m_a \leq m_b \leq m_c$$

$$\sum m_a \csc A = 2R \sum \frac{m_a}{a} \overset{Chebyshev}{\geq} 2R \cdot \frac{1}{3}\left(\sum m_a\right)\left(\sum \frac{1}{a}\right) \geq$$

$$\geq 2R\frac{1}{3}\left(\sum h_a\right)\left(\sum \frac{1}{a}\right) \overset{Leuenberger\ \&\ (1)}{\geq} 2R\frac{1}{3}(9r)\frac{\sqrt{3}}{R} = 6\sqrt{3}r$$

$$Equality\ for\ a = b = c$$