ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{\tan^2 A + \tan^2 B}{\tan^4 A + \tan^4 B} \le 1$$

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Solution by Tapas Das-India

$$A + B + C = \pi \text{ or } A + B = \pi - A \text{ or } \tan(A + B) = -\tan C$$

$$\text{or } \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\text{or } \sum \tan A = \prod \tan A \ \ (1) \text{ and}$$

$$\frac{\tan^2 A + \tan^2 B}{\tan^4 A + \tan^4 B} \overset{CBS}{\le} \frac{\tan^2 A + \tan^2 B}{\frac{(\tan^2 A + \tan^2 B)^2}{2}} = \frac{2}{\tan^2 A + \tan^2 B} \overset{AM-GM}{\le}$$

$$\le \frac{2}{2\tan A \tan B} = \frac{1}{\tan A \tan B} (1)$$

$$\sum \frac{\tan^2 A + \tan^2 B}{\tan^4 A + \tan^4 B} \overset{(1)}{\le} \sum \frac{1}{\tan A \tan B} = \frac{\sum \tan A}{\prod \tan A} \overset{(1)}{=} 1$$

$$\text{Equality for } A = B = C$$