

# ROMANIAN MATHEMATICAL MAGAZINE

In acute  $\triangle ABC$  holds:

$$\sum \sin A + \sum \tan A \geq \frac{9\sqrt{3}}{2}$$

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*Solution by Tapas Das-India*

Let be  $f(x) = \sin x + \tan x, x \in \left(0, \frac{\pi}{2}\right)$

$f''(x) = \sin x (2 \sec^3 x - 1) > 0$  as  $\sec x > 1$ ,  $f$  is convex.

$$f(A) + f(B) + f(C) \stackrel{\text{Jensen}}{\geq} 3f\left(\frac{A+B+C}{3}\right) = 3f\left(\frac{\pi}{3}\right) =$$

$$= 3\left(\sin\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)\right) = 3\left(\frac{\sqrt{3}}{2} + \sqrt{3}\right) = \frac{9\sqrt{3}}{2}$$

$$\sum \sin A + \sum \tan A \geq \frac{9\sqrt{3}}{2}$$

Equality for  $A = B = C = \frac{\pi}{3}$