

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\sum \frac{a}{r_a} \geq 2\sqrt{3}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum \frac{a}{r_a} &= \frac{1}{F} \sum a(s-a) = \frac{1}{F} (2s^2 - \sum a^2) = \\ &= \frac{1}{F} (2s^2 - 2s^2 + 2r^2 + 8Rr) = \frac{2r(4R+r)}{rs} = \frac{2(4R+r)}{s} \stackrel{\text{Doucet}}{\geq} 2\sqrt{3} \end{aligned}$$

Equality for $a = b = c$