ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{h_b+h_c}+\frac{b}{h_c+h_a}+\frac{c}{h_a+h_b}\geq\sqrt{3}$$

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Solution by Daniel Sitaru-Romania

$$\frac{a}{h_b+h_c}+\frac{b}{h_c+h_a}+\frac{c}{h_a+h_b}=\sum_{cyc}\frac{a}{h_b+h_c}=\sum_{cyc}\frac{a}{\frac{2F}{b}+\frac{2F}{c}}=$$

$$=\frac{1}{2F}\sum_{cyc}\frac{a}{\frac{1}{b}+\frac{1}{c}}=\frac{1}{2F}\sum_{cyc}\frac{abc}{b+c}=\frac{abc}{2F}\sum_{cyc}\frac{1}{b+c}\geq$$

$$\overset{BERGSTROM}{\geq}\frac{4RF}{2F}\cdot\frac{(1+1+1)^2}{b+c+c+a+a+b}=2R\cdot\frac{9}{2(a+b+c)}=$$

$$=\frac{9R}{2s}\overset{MITRINOVIC}{\geq}\frac{9R}{2\cdot\frac{3\sqrt{3}}{2}\cdot R}=\frac{3}{\sqrt{3}}=\sqrt{3}$$

Equality holds for $a=b=c$.