ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{r_b+r_c}+\frac{b}{r_c+r_a}+\frac{c}{r_a+r_b}\geq\sqrt{3}$$

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Solution by Daniel Sitaru-Romania

$$\frac{a}{r_b+r_c}+\frac{b}{r_c+r_a}+\frac{c}{r_a+r_b}=\sum_{cyc}\frac{a}{r_b+r_c}=\sum_{cyc}\frac{a}{\frac{F}{s-b}+\frac{F}{s-c}}=$$

$$=\frac{1}{F}\sum_{cyc}\frac{a}{\frac{1}{s-b}+\frac{1}{s-c}}=\frac{1}{F}\sum_{cyc}\frac{a(s-b)(s-c)}{s-c+s-b}=\frac{1}{rs}\sum_{cyc}\frac{a(s-b)(s-c)}{a}=$$

$$=\frac{1}{rs}\sum_{cyc}(s-b)(s-c)=\frac{1}{rs}\cdot(4R+r)r=\frac{4R+r}{s}\overset{DOUCET}{\geq}\frac{s\sqrt{3}}{s}=\sqrt{3}$$

Equality holds for: $a=b=c$.