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In ΔABC the following relationship holds:

$$(a+b+c)\left(\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}\right)\ge 6\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$(a+b+c)\left(\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}\right)=2s\left(\frac{s-a}{F}+\frac{s-b}{F}+\frac{s-c}{F}\right)=$$

$$=\frac{2s}{F}(3s-a-b-c)=\frac{2s}{rs}(3s-2s)=\frac{2s}{r}\ge$$

$$\overset{MITRINOVIC}{\ge}\frac{2\cdot 3\sqrt{3}r}{r}=6\sqrt{3}$$

Equality holds for $a=b=c$.