ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{h_a}+\frac{b}{h_b}+\frac{c}{h_c}\le\frac{\sqrt{3}R^2}{2r^2}$$

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Solution by Daniel Sitaru-Romania

$$\frac{a}{h_a}+\frac{b}{h_b}+\frac{c}{h_c}=\sum_{cyc}\frac{a}{h_a}=\sum_{cyc}\frac{a}{\frac{2F}{a}}=\frac{1}{2F}\sum_{cyc}a^2=$$

$$=\frac{1}{2F}\cdot 2(s^2-r^2-4Rr)\overset{GERRETSEN}{\overbrace{\le}}\frac{1}{F}\cdot(4R^2+4Rr+3r^2-r^2-4Rr)=$$

$$=\frac{4R^2+2r^2}{rs}\overset{MITRINOVIC}{\overbrace{\le}}\frac{4R^2+2r^2}{r\cdot 3\sqrt{3}r}\overset{EULER}{\overbrace{\le}}\frac{4R^2+2\cdot\frac{R^2}{4}}{3\sqrt{3}r^2}=$$

$$=\frac{1}{3\sqrt{3}r^2}\cdot\frac{9R^2}{2}=\frac{\sqrt{3}R^2}{2r^2}$$

Equality holds for $a=b=c$.