ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a^2}{m_b^2+m_c^2}+\frac{b^2}{m_c^2+m_a^2}+\frac{c^2}{m_a^2+m_b^2}\ge 2$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$WLOG\ a\ge b\ge c\ then\ m_a\le m_b\le\ m_c\ and$$

$$m_a^2+m_b^2\le m_c^2+m_a^2\le m_b^2+m_c^2$$

$$\frac{1}{m_a^2+m_b^2}\ge\frac{1}{m_c^2+m_a^2}\ge\frac{1}{m_b^2+m_c^2}$$

$$\frac{a^2}{m_b^2+m_c^2}+\frac{b^2}{m_c^2+m_a^2}+\frac{c^2}{m_a^2+m_b^2}\overset{Chebyshev}{\ge}\frac{1}{3}\left(\sum a^2\right)\left(\sum\frac{1}{m_b^2+m_c^2}\right)\overset{Bergstrom}{\ge}$$

$$\ge\frac{1}{3}\cdot\frac{(\sum a^2)(1+1+1)^2}{2(\sum m_a^2)}=\frac{1}{3}\cdot\frac{(\sum a^2)(3)^2}{2\left(\frac{3}{4}\sum a^2\right)}=2$$

$$Equality\ for\ a=b=c$$