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In $\Delta ABC, I$ −incenter, the following relationship holds:

$$\sqrt{abc - aIA^2} + \sqrt{abc - bIB^2} + \sqrt{abc - cIC^2} \le \sqrt{6abc}$$

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Solution by Tapas Das-India

$$\sum aIA^2 = \sum \left(\frac{abc(s-a)}{s} \right) = abc \sum \frac{s-a}{s} = abc \ (1)$$

$$\sqrt{abc - aIA^2} + \sqrt{abc - bIB^2} + \sqrt{abc - cIC^2} \overset{CBS}{\le}$$

$$\le \sqrt{3\left(3abc - \sum aIA^2\right)} \overset{(1)}{=} \sqrt{\left(3(3abc - abc)\right)} = \sqrt{6abc}$$

$$Equality\ holds\ for\ a = b = c$$