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Prove that in any acute ΔABC:

$$\cos^2 A + \cos^2 B + \cos^2 C + 6\cos A\cos B\cos C \le \cos A + \cos B + \cos C$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

The desired inequality can be rewritten as

$$\sum_{cyc}(1-\cos A-2\cos B\cos C)\cos A \ge 0,$$

which is true because $\cos A > 0$ **(and analogs), and**

$$1-\cos A-2\cos B\cos C =$$

$$= 1-\cos A-\cos(B+C)-\cos(B-C) \overset{B+C=\pi-A}{\cong} 1-\cos(B-C) \ge 0 \text{ (and analogs)}.$$

So the proof is complete. Equality holds if ΔABC **is equilateral.**