ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a}{h_a}+\frac{r_b}{h_b}+\frac{r_c}{h_c}\ge 3$$

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Solution by Daniel Sitaru-Romania

$$\frac{r_a}{h_a}+\frac{r_b}{h_b}+\frac{r_c}{h_c}=\sum_{cyc}\frac{r_a}{h_a}=\sum_{cyc}\frac{\frac{F}{s-a}}{\frac{2F}{a}}=\frac{1}{2}\sum_{cyc}\frac{a}{s-a}=$$

$$=\frac{1}{2(s-a)(s-b)(s-c)}\sum_{cyc}a(s-b)(s-c)=$$

$$=\frac{s}{2s(s-a)(s-b)(s-c)}\cdot 2rs(2R-r)=\frac{s^2r(2R-r)}{F^2}=$$

$$=\frac{s^2r(2R-r)}{s^2r^2}=\frac{2R-r}{r}\overset{EULER}{\ge}\frac{4r-r}{r}=3$$

Equality holds for: $a=b=c$.