ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC holds:

$$\frac{a}{h_a}+\frac{b}{h_b}+\frac{c}{h_c}\ge 2\left(\tan\frac{A}{2}+\tan\frac{B}{2}+\tan\frac{C}{2}\right)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum\frac{a}{h_a}=\frac{\sum a^2}{2F}=\frac{2(s^2-4Rr-r^2)}{2rs}\overset{Gerretsen}{\ge}\frac{12Rr-6r^2}{2rs}$$

$$2\sum\tan\frac{A}{2}=\frac{2(4R+r)}{s}$$

$$We\ need\ to\ show:$$

$$\frac{12Rr-6r^2}{2rs}\ge\frac{2(4R+r)}{s}\ or\ 4Rr-8r^2\ge 0\ or\ R\ge 2r(Euler)$$