ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{\cot\frac{A}{2}}{\tan\frac{B}{2}+\tan\frac{C}{2}} \geq \frac{9}{2}$$

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Solution by Tapas Das-India

$$\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}\ or \tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)\ or,$$

$$\frac{\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\cot\frac{C}{2}\ or \sum \tan\frac{A}{2}\tan\frac{B}{2}=1\ (1)$$

$$\sum \frac{\cot\frac{A}{2}}{\tan\frac{B}{2}+\tan\frac{C}{2}}=\sum \frac{1^2}{\tan\frac{B}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{C}{2}} \overset{Bergstrom}{\geq} \frac{(1+1+1)^2}{2\sum \tan\frac{A}{2}\tan\frac{B}{2}} \overset{(1)}{=} \frac{9}{2}$$

$$Equality\ holds\ A=B=C=\frac{\pi}{3}$$