ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{l_a}+\frac{b}{l_b}+\frac{c}{l_c}\geq 2\sqrt{3}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\boxed{l_a=\frac{2bc}{b+c}.\cos\frac{A}{2}}$$

$$\frac{a}{l_a}+\frac{b}{l_b}+\frac{c}{l_c}=\frac{a(b+c)}{2bc.\cos\frac{A}{2}}+\frac{b(a+c)}{2ac.\cos\frac{B}{2}}+\frac{c(a+b)}{2ab.\cos\frac{C}{2}}\overset{A-G}{\geq}$$

$$\geq\frac{a\sqrt{bc}}{bc.\cos\frac{A}{2}}+\frac{b\sqrt{ac}}{ac.\cos\frac{B}{2}}+\frac{c\sqrt{ab}}{ab.\cos\frac{C}{2}}\overset{A-G}{\geq}$$

$$\geq 3\left(\frac{a^2b^2c^2}{a^2b^2c^2.\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}}\right)^{\frac{1}{3}}=3\left(\frac{4R}{P}\right)^{\frac{1}{3}}\geq 3\left(4.\ \frac{2}{3\sqrt{3}}\right)^{\frac{1}{3}}=2\sqrt{3}$$

Equality holds if the triangle is an equilateral one.