ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$m_a^{r_a} m_b^{r_b} m_c^{r_c} \le \left(\frac{3R}{2}\right)^{\frac{9R}{2}}$$

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$$WLOG\ a \ge b \ge c\ then\ m_a \le m_b \le m_c\ and\ r_a \ge r_b \ge r_c$$

$$m_a r_a + m_b r_b + m_c r_c \overset{Chebyshev}{\le} \frac{1}{3}\left(\sum m_a\right)\left(\sum r_a\right)\ (1)$$

$$and\ \left(\sum r_a\right) = 4R + r \overset{Euler}{\le} \frac{9R}{2}(2), \left(\sum m_a\right) \overset{Leunberger}{\le} 4R + r \overset{Euler}{\le} \frac{9R}{2}(3)$$

$$\left(m_a^{r_a} m_b^{r_b} m_c^{r_c}\right)^{\frac{1}{(\sum r_a)}} \overset{AM-Gm}{\le} \frac{m_a r_a + m_b r_b + m_c r_c}{(\sum r_a)} \overset{(1)}{\le} \frac{\frac{1}{3}(\sum m_a)(\sum r_a)}{(\sum r_a)} =$$

$$= \frac{1}{3}\left(\sum m_a\right) \overset{(3)}{\le} \frac{1}{3}\frac{9R}{2} = \frac{3R}{2}$$

$$or\ m_a^{r_a} m_b^{r_b} m_c^{r_c} \le \left(\frac{3R}{2}\right)^{(\sum r_a)} \overset{(2)}{\le} \left(\frac{3R}{2}\right)^{\frac{9R}{2}}$$

$$Equality\ holds\ for\ a = b = c$$