ROMANIAN MATHEMATICAL MAGAZINE

In all acute ΔABC the following relationship holds:

$$\frac{ca+cb}{a^2+b^2-c^2}+\frac{ab+ac}{b^2+c^2-a^2}+\frac{bc+ba}{c^2+a^2-b^2}\geq 6$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{ca+cb}{a^2+b^2-c^2}+\frac{ab+ac}{b^2+c^2-a^2}+\frac{bc+ba}{c^2+a^2-b^2}=\sum_{cyc}\frac{ca+cb}{a^2+b^2-c^2}=$$

$$=\sum_{cyc}\frac{c(a+b)}{2abcosC}\overset{AM-GM}{\geq}\sum_{cyc}\frac{c\cdot 2\sqrt{ab}}{2abcosC}\overset{AM-GM}{\geq}$$

$$\geq 3\cdot\sqrt[3]{\frac{abc\cdot\sqrt{ab}\cdot\sqrt{bc}\cdot\sqrt{ca}}{ab\cdot bc\cdot ca\cdot cosAcosBcosC}}=3\sqrt[3]{\frac{1}{cosAcosBcosC}}\geq$$

$$\geq 3\cdot\sqrt[3]{\frac{1}{\frac{1}{8}}}=3\cdot\sqrt[3]{8}=6$$

Equality holds for $a=b=c$.