ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{(sinB+sinC)^2}+\frac{b}{(sinC+sinA)^2}+\frac{c}{(sinA+sinB)^2}\geq\sqrt{3}R$$

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$$\frac{a}{(sinB+sinC)^2}=\frac{2RsinA}{4sin^2\frac{B+C}{2}\,.\,cos^2\frac{B-C}{2}}=\frac{4Rsin\frac{A}{2}\,.\,cos\frac{A}{2}}{4cos^2\frac{A}{2}\,.\,cos^2\frac{B-C}{2}}=$$

$$\frac{R\,.\,tan\frac{A}{2}}{cos^2\frac{B-C}{2}}\geq R\,.\,tan\frac{A}{2}\quad\left(Because\ cos^2\frac{B-C}{2}\leq 1\right)$$

$$So\ \ \frac{a}{(sinB+sinC)^2}\geq R\,.\,tan\frac{A}{2}$$

Analogously:

$$\frac{b}{(sinC+sinA)^2}\geq R\,.\,tan\frac{B}{2}\ ;\ \frac{c}{(sinA+sinB)^2}\geq R\,.\,tan\frac{C}{2}$$

Let's summarize the results :

$$\frac{a}{(sinB+sinC)^2}+\frac{b}{(sinC+sinA)^2}+\frac{c}{(sinA+sinB)^2}\geq R\,.\left(tan\frac{A}{2}+tan\frac{B}{2}+tan\frac{C}{2}\right)\geq\sqrt{3}R$$

$$But\ \ \boxed{\left(tan\frac{A}{2}+tan\frac{B}{2}+tan\frac{C}{2}\right)\geq\sqrt{3}}\ \ (true)$$

Equality holds iff $a=b=c$