ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1}{R}+\frac{1}{r}\geq\frac{9\sqrt{3}}{2s}$$

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Solution by Tapas Das-India

$$We\ need\ to\ show\ \frac{1}{R}+\frac{1}{r}\geq\frac{9\sqrt{3}}{2s}\ \ or\ 2s(R+r)\geq 9\sqrt{3}Rr\ or$$

$$4s^2(R+r)^2\geq 243\ R^2r^2$$

$$4(16Rr-5r^2)(R+r)^2\geq 243R^2r^2(Gerretsen)\ or$$

$$4\left(\frac{16R}{r}-5\right)\left(\frac{R}{r}+1\right)^2\geq 243\left(\frac{R}{r}\right)^2\ \ or$$

$$4(16x-5)(x+1)^2\geq 243x^2\left(\frac{R}{r}=x\geq 2\ Euler\right)\ or$$

$$4(16x-5)(x^2+2x+1)\geq 243x^2$$

$$4(16x^3+27x^2+6x-5)\geq 243x^2\ or$$

$$64x^3-135x^2+24x-20\geq 0\ or$$

$$(x-2)(64x^2-7x+10)\geq 0\ true\ as\ x\geq 2$$

$$Equality\ holds\ \ for\ an\ \ equilateral\ triangle$$