ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1}{\sin A}+\frac{1}{\sin B}+\frac{1}{\sin C}\le\frac{2}{3}\left(\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}\right)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum\frac{1}{\sin A}=2R\sum\frac{1}{a}=\frac{2R(ab+bc+ca)}{abc}=\frac{s^2+r^2+4Rr}{2rs}\ and$$

$$\frac{2}{3}\sum\cot\frac{A}{2}=\frac{2}{3}\frac{s}{r}$$

$$We\ need\ to\ show:$$

$$\frac{2s}{3r}\ge\frac{s^2+r^2+4Rr}{2rs}\ or\ 4s^2\ge 3s^2+3r^2+12Rr$$

$$or\ s^2\ge 3r^2+12Rr\ or\ 16Rr-5r^2\overset{Gerretsen}{\ge}12Rr+3r^2$$

$$or\ R\ge 2r\ (Euler)$$