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In $\triangle ABC$ the following relationship holds:

$$\frac{a^2}{(b+c)^2 - a^2} + \frac{b^2}{(c+a)^2 - b^2} + \frac{c^2}{(a+b)^2 - c^2} \geq 1$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \sum_{cyc} \frac{a^2}{(b+c)^2 - a^2} &= \sum_{cyc} \frac{a^2}{(b+c+a)(b+c-a)} = \\ &= \sum_{cyc} \frac{a^2}{2s(2s-2a)} = \frac{1}{4} \sum_{cyc} \frac{a^2}{s(s-a)} \stackrel{\text{BERGSTROM}}{\geq} \\ &\geq \frac{1}{4} \cdot \frac{(a+b+c)^2}{s(s-a) + s(s-b) + s(s-c)} = \\ &= \frac{1}{4} \cdot \frac{4s^2}{s(s-a) + s(s-b) + s(s-c)} = \frac{s}{3s-a-b-c} = \frac{s}{3s-2s} = 1 \end{aligned}$$

Equality holds for $a = b = c$.