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In ΔABC the following relationship holds:

$$\frac{R^2+6Rr-\sqrt{R(R-2r)^3}}{4R^2}\le\prod\cos\frac{A-B}{2}\le\frac{R^2+6Rr+\sqrt{R(R-2r)^3}}{4R^2}$$

Proposed by Nguyen Minh Tho-Vietnam

Solution by Tapas Das-India

Blundon's inequality:

$$2R^2+10Rr-r^2-2\sqrt{R(R-2r)^3}\le s^2\le 2R^2+10Rr-r^2+2\sqrt{R(R-2r)^3}\ (1)$$

$$\prod\cos\frac{A-B}{2}=\frac{s^2+r^2+2Rr}{8R^2}\overset{(1)}{\ge}\frac{2R^2+10Rr-r^2-2\sqrt{R(R-2r)^3}+r^2+2Rr}{8R^2}=$$

$$=\frac{2R^2+12Rr-2\sqrt{R(R-2r)^3}}{8R^2}=\frac{R^2+6Rr-\sqrt{R(R-2r)^3}}{4R^2}$$

$$\prod\cos\frac{A-B}{2}=\frac{s^2+r^2+2Rr}{8R^2}\overset{(1)}{\le}\frac{2R^2+10Rr-r^2+2\sqrt{R(R-2r)^3}+r^2+2Rr}{8R^2}=$$

$$=\frac{2R^2+12Rr+2\sqrt{R(R-2r)^3}}{8R^2}=\frac{R^2+6Rr+\sqrt{R(R-2r)^3}}{4R^2}$$

Equality holds for an equilateral triangle