ROMANIAN MATHEMATICAL MAGAZINE

Solve the differential equation:

$$u_{tt} + u_{xx} = 0 \quad u(x,0) = \sec^a x + cosec^b x - 1 \qquad u_t(x\,,0) = 0$$

where: b+1=a=tg2z-2 and $\frac{2}{tg2x} = sinzcosz(2ctgz - 1), 0 < x < \frac{\pi}{2}$

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Solution by proposer

Firstly, solve the trigonometric equation: $0 < x < \frac{\pi}{2}$

$$\frac{2}{tg2x} = \frac{2sinxcosxcosx}{sinx} - sinxcosx$$

$and\ accepted\ that$,

$$tg2x = \frac{2tgx}{1 - tg^2x} \quad sinx = cosxtgx \quad cosx = \frac{1}{1 + tg^2x}.Then:$$

$$\frac{1 - tg^2x}{tgx} = \frac{2 - tgx}{1 + tg^2x}$$

$$1 - tg^4x = 2tgx - tg^2x$$

$$tg^4x - tg^2x + 2tgx - 1 = 0$$

$$(tg^2x)^2 - (tgx - 1)^2 = 0$$

$$tg^2x - tgx + 1 = 0\ here\ D < 0 \quad but \quad tg^2x + tgx - 1 = 0$$

$$tgx = \frac{\sqrt{5} - 1}{2}$$

According to the formula above: $tg2x = 2\,.And\ a = 0\,,\ b = -1$

We consider in wave equation:

$$u_{tt} + u_{xx} = 0 \quad u(x,0) = sinx \qquad u_t(x\,,0) = 0$$

$$u(x,t) = \frac{sin(x-it)+sin(x+it)}{2} = \frac{\frac{e^{i(x-it)}-e^{-i(x-it)}}{2i}+\frac{e^{i(x+it)}-e^{-i(x+it)}}{2i}}{2} = \frac{e^{ix}-e^{-ix}}{2i}\frac{e^{t}+e^{-t}}{2} = sinxcht$$