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If $x,y>0, k\in\mathbb{N}^*$ then:

$$\sum_{n=1}^{k}\left(\sum_{m=1}^{n}\left(\frac{m}{\sqrt{x}}+\frac{\sqrt{y}}{n^2}\right)\sqrt{\frac{x}{m^2}+\frac{n^4}{y}}\right)\geq\sqrt{2}k(k+1)$$

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Solution by Daniel Sitaru-Romania

$$\sum_{n=1}^{k}\left(\sum_{m=1}^{n}\left(\frac{m}{\sqrt{x}}+\frac{\sqrt{y}}{n^2}\right)\sqrt{\frac{x}{m^2}+\frac{n^4}{y}}\right)\overset{AM-GM}{\geq}\sum_{n=1}^{k}\left(\sum_{m=1}^{n}\left(\frac{m}{\sqrt{x}}+\frac{\sqrt{y}}{n^2}\right)\sqrt{2\sqrt{\frac{x}{m^2}\cdot\frac{n^4}{y}}}\right)=$$

$$=\sum_{n=1}^{k}\left(\sum_{m=1}^{n}\left(\frac{m}{\sqrt{x}}+\frac{\sqrt{y}}{n^2}\right)\sqrt{2}\cdot\sqrt{\frac{\sqrt{x}}{m}\cdot\frac{n^2}{\sqrt{y}}}\right)\overset{AM-GM}{\geq}$$

$$\geq 2\sqrt{2}\sum_{n=1}^{k}\left(\sum_{m=1}^{n}\sqrt{\frac{m}{\sqrt{x}}\cdot\frac{\sqrt{y}}{n^2}\cdot\frac{\sqrt{x}}{m}\cdot\frac{n^2}{\sqrt{y}}}\right)=2\sqrt{2}\sum_{n=1}^{k}\left(\sum_{m=1}^{n}1\right)=$$

$$=2\sqrt{2}\sum_{n=1}^{k}n=2\sqrt{2}\cdot\frac{k(k+1)}{2}=\sqrt{2}k(k+1)$$