ROMANIAN MATHEMATICAL MAGAZINE

If $a=\frac{\varphi^2}{2(1+\varphi)}, b=\frac{\varphi}{2(1+\varphi)}, c=\frac{1}{2(1+\varphi)}, \varphi-$ golden ratio, then

$$\sum_{cyc}\frac{a^2b^2}{c^3(a^2-ab+b^2)}>6\varphi$$

Proposed by Khaled Abd Imouti-Syria

Solution by Izumi Ainsworth-Japan

$$a=\frac{\varphi^2}{2(\varphi+1)}=\frac{1}{2}, b=\frac{\varphi}{2(\varphi+1)}=\frac{1}{2\varphi}, c=\frac{1}{2(1+\varphi)}=\frac{1}{2\varphi^2}$$

$$*\ \frac{a^2b^2}{c^3(a^2-ab+b^2)}=\frac{\left(\frac{1}{2\varphi}\right)^2\left(\frac{1}{2\varphi^2}\right)^2}{\left(\frac{1}{2\varphi}\right)^3\left(\frac{1}{4}-\frac{1}{4\varphi}+\frac{1}{4\varphi^2}\right)}=\frac{2\varphi^4}{\frac{2}{\varphi^2}}=\varphi^6\quad(\alpha)$$

$$*\ \frac{b^2c^2}{a^3(b^2-bc+c^2)}=\frac{\left(\frac{1}{2\varphi}\right)^2\left(\frac{1}{2\varphi^2}\right)^2}{\left(\frac{1}{2}\right)^3\left(\frac{1}{4\varphi^2}-\frac{1}{4\varphi^3}+\frac{1}{4\varphi^4}\right)}=\frac{2}{\varphi^6\left(\frac{2}{\varphi^4}\right)}=\frac{1}{\varphi^2}\quad(\beta)$$

$$*\ \frac{c^2a^2}{b^3(c^2-ca+a^2)}=\frac{\left(\frac{1}{2\varphi^2}\right)^2\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2\varphi}\right)^3\left(\frac{1}{4\varphi^4}-\frac{1}{4\varphi^4}+\frac{1}{4}\right)}=\frac{2}{\varphi\left(\frac{2}{\varphi^2}\right)}=\frac{1}{\varphi}\quad(\gamma)$$

$$(\alpha)+(\beta)+(\gamma):$$

$$\therefore\sum_{cyc}\frac{a^2b^2}{c^3(a^2-ab+b^2)}=\varphi^6+\frac{1}{\varphi^2}+\frac{1}{\varphi}$$

$$=(8\varphi+5)+(2-\varphi)+(\varphi-1)=8\varphi+6>6\varphi$$

$$\varphi\to\frac{\sqrt{5}+1}{2}\quad\text{(Golden ratio)}$$