ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ and $5a^2 + 4b^2 + 3c^2 + 2abc = 60$, then prove that :

$$a + b + c \leq \sqrt{60}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Soumava Chakraborty-Kolkata-India

Let us assume $a + b + c > \sqrt{60}$ and $60 = 5a^2 + 4b^2 + 3c^2 + 2abc$

$$= (3a^2 + 3b^2 + 3c^2) + 2a^2 + b^2 + 2abc \geq (a + b + c)^2 + 2a^2 + b^2 + 2abc$$

$$\overset{\text{assumption}}{>} 60 + 2a^2 + b^2 + 2abc \Rightarrow 2a^2 + b^2 + 2abc < 0 \rightarrow \textit{impossible}$$

$\because a, b, c > 0 \therefore$ our assumption is incorrect and hence we conclude that :

$$a + b + c \leq \sqrt{60} \text{ (QED)}$$