ROMANIAN MATHEMATICAL MAGAZINE

If $x,y>0,\ 3^x+3^y=6$ then:

$$(2x^2+y)(2y^2+x)+9xy\le 18$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$3^x+3^y=6\ or\ \frac{3^x+3^y}{2}=3\ or\ \sqrt{3^{x+y}}\overset{AM-GM}{\le}3\ or\ 3^{\frac{x+y}{2}}\le 3\ or$$

$$\frac{x+y}{2}\le 1\ or\ x+y\le 2 \qquad (1)$$

$$4x^2y^2=4xy.(xy)\overset{AM-GM}{\le}4xy\frac{(x+y)^2}{4}=xy(x+y)^2\overset{(1)}{\le}4xy \qquad (2)$$

$$x^3+y^3=(x+y)(x^2+y^2-xy)\overset{(1)}{\le}2(x^2+y^2-xy) \qquad (3)$$

$$(2x^2+y)(2y^2+x)+9xy=4x^2y^2+2(x^3+y^3)+10xy\overset{(2)\&(3)}{\le}$$

$$\le 4xy+4(x^2+y^2-xy)+6xy=$$

$$=4(x+y)^2+2xy\overset{(1)\&\,AM-GM}{\le}4\cdot 2^2+\frac{2(x+y)^2}{4}\overset{(1)}{\le}16+\frac{2^2}{2}=18$$

Equality holds for x=y=1.