ROMANIAN MATHEMATICAL MAGAZINE

$$\Omega(x)=\Gamma\left(\frac{x}{2}\right).\Gamma\left(\frac{x+1}{2}\right).\Gamma\left(\frac{1-x}{2}\right).\Gamma\left(\frac{2-x}{2}\right).\sin(\pi x)$$

Solve for real numbers:

$$x^2-\frac{4x}{\Omega(x)}+\frac{1}{\pi^4}=0$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\Omega(x)=\Gamma\left(\frac{x}{2}\right).\Gamma\left(\frac{x+1}{2}\right).\Gamma\left(\frac{1-x}{2}\right).\Gamma\left(\frac{2-x}{2}\right).\sin(\pi x)$$

$$Note:\ \Gamma(x).\Gamma(1-x)=\frac{\pi}{\sin(\pi x)}$$

$$\Gamma\left(\frac{x}{2}\right).\Gamma\left(\frac{2-x}{2}\right)=\Gamma\left(\frac{x}{2}\right).\Gamma\left(1-\frac{x}{2}\right)=\frac{\pi}{\sin(\frac{\pi}{2}x)}$$

$$\Gamma\left(\frac{x+1}{2}\right).\Gamma\left(\frac{1-x}{2}\right)=\Gamma\left(\frac{x+1}{2}\right).\Gamma\left(1-\frac{x+1}{2}\right)=\frac{\pi}{\sin(\frac{\pi}{2}-\frac{\pi}{2}x)}=\frac{\pi}{\cos(\frac{\pi}{2}x)}$$

$$\Omega(x)=\Gamma\left(\frac{x}{2}\right).\Gamma\left(\frac{x+1}{2}\right).\Gamma\left(\frac{1-x}{2}\right).\Gamma\left(\frac{2-x}{2}\right).\sin(\pi x)$$

$$=\frac{\pi}{\sin(\frac{\pi}{2}x)}.\frac{\pi}{\cos(\frac{\pi}{2}x)}.\sin(\pi x)=$$

$$\frac{2\pi^2}{\sin(2.\frac{\pi}{2}x)}.\sin(\pi x)=2\pi^2$$

$$x^2-\frac{4x}{\Omega(x)}+\frac{1}{\pi^4}=0\rightarrow x^2-\frac{4x}{2\pi^2}+\frac{1}{\pi^4}=0\rightarrow\left(x-\frac{1}{\pi^2}\right)^2=0$$

$$Answer:\ x=\frac{1}{\pi^2}$$