ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega_{n,p,t}=\int_a^b \frac{ln^n(x+1)}{x^p(x+1)^t}dx \ \ \{0\le p,t,n\in Z \ and \ 0\le a<b\}$$

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$$\int_a^b \frac{ln^n(x+1)}{x^p(x+1)^t}dx \overset{1+x=e^z}{\underset{dx=e^z dz}{=}} \int_{\ln(1+a)}^{\ln(1+b)} \frac{z^n e^{-z(t-1+p)}}{(1-e^{-z})^p}dz$$

$$\frac{1}{(1-y)^m}=\sum_{k=m-1}^{\infty} C_k^{m-1}\, y^{k+1-m}$$

$$\sum_{k=p-1}^{\infty} C_k^{p-1}\int_{\ln(1+a)}^{\ln(1+b)} z^n\, e^{-z(t+k)}dz \overset{z=\frac{y}{t+k}}{\cong}$$

$$\sum_{k=p-1}^{\infty}\frac{C_k^{p-1}}{(t+k)^{n+1}}\int_{(t+k)\ln(1+a)}^{(t+k)\ln(1+b)} y^n\, e^{-y}dy \overset{e^{-y}=\sum_{i=0}^{\infty}\frac{(-1)^i y^i}{i!}}{\cong}$$

$$\sum_{k=p-1}^{\infty}\frac{C_k^{p-1}}{(t+k)^{n+1}}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\int_{(t+k)\ln(1+a)}^{(t+k)\ln(1+b)} y^{n+i}\, dy=$$

$$\sum_{k=p-1}^{\infty}\frac{C_k^{p-1}}{(t+k)^{n+1}}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\,(n+i+1)}(t+k)^{n+i+1}[ln^{n+i+1}(1+b)-ln^{n+i+1}(1+a)]$$