ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_1^{\sqrt{2}} \frac{dx}{x\sqrt{-x^4+3x^2-2}}$$

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$$\int_1^{\sqrt{2}} \frac{dx}{x\sqrt{-x^4+3x^2-2}} \overset{x\to\frac{1}{t}}{\cong} \int_1^{\frac{1}{\sqrt{2}}} \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{-\frac{1}{t^4}+\frac{3}{t^2}-2}} = \int_{\frac{1}{\sqrt{2}}}^1 \frac{tdt}{\sqrt{-2t^4+3t^2-1}} =$$

$$= \frac{1}{2}\int_{\frac{1}{\sqrt{2}}}^1 \frac{dt^2}{\sqrt{-2t^4+3t^2-1}} \overset{u\to t^2}{\cong} \frac{1}{2}\int_{\frac{1}{2}}^1 \frac{du}{\sqrt{-2u^2+3u-1}} \overset{v=-4u+3}{\cong}$$

$$\left\{v=-4u+3 \to u=\frac{3-v}{4} \to du=-\frac{1}{4}dv\right\}$$

$$= -2u^2+3u-1 = -2.\frac{(3-v)^2}{16}+3.\frac{3-v}{4}-1 = -\frac{(9-6v+v^2)}{8}+\frac{9-3v}{4}-1 =$$

$$= \frac{-9-v^2+6v+18-6v-8}{8} = \frac{1-v^2}{8} = \frac{1}{2}\int_1^{-1} \frac{-\frac{1}{4}dv}{\sqrt{\frac{1-v^2}{8}}} = \frac{1}{2}.\left(\frac{1}{4}\right).\sqrt{8}\int_{-1}^1 \frac{dv}{\sqrt{1-v^2}} =$$

$$= \frac{\sqrt{2}}{4}.2\int_0^1 \frac{dv}{\sqrt{1-v^2}} = \frac{\sqrt{2}}{2}(\arcsin(v)|_0^1 = \frac{\sqrt{2}}{2}(\arcsin(1)-\arcsin(0)) = \frac{\pi\sqrt{2}}{4}\,(proved)$$

$$\int_1^{\sqrt{2}} \frac{dx}{x\sqrt{-x^4+3x^2-2}} = \frac{\pi\sqrt{2}}{4}$$