

# ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_1^2 \frac{x\sqrt{x-1}}{x-5} dx$$

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$$\int_1^2 \frac{x\sqrt{x-1}}{x-5} dx \stackrel{\sqrt{x-1} \rightarrow t}{=} \int_0^1 \frac{2t^2(t^2+1)}{t^2-4} dt = 2 \int_0^1 \frac{t^4+t^2}{t^2-4} dt =$$

$$\text{Let : } \sqrt{x-1} \rightarrow t \Rightarrow x = t^2 + 1 \Rightarrow dx = 2tdt$$

$$= 2 \left( \int_0^1 \left( t^2 + 5 - \frac{20}{4-t^2} \right) dt = 2 \left( \left( \frac{t^3}{3} + 5t \right) \Big|_0^1 - 20 \cdot \frac{1}{4} \ln \left| \frac{2+t}{2-t} \right| \Big|_0^1 \right) =$$

$$= 2 \left( \left( \frac{1}{3} + 5 \right) - 5(\ln(3) - \ln(1)) \right) = 2 \left( \frac{16}{3} - 5 \ln(3) \right) = \frac{32}{3} - 10 \ln(3)$$

$$\int_1^2 \frac{x\sqrt{x-1}}{x-5} dx = \frac{32}{3} - 10 \ln(3) \text{ (proved)}$$