ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_{e^2}^{e^3} \frac{\ln(x)-4}{\left(1-ln^2(x)\right)}dx$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Arowolo Isaiah-Nigeria

$$\int_{e^2}^{e^3} \frac{\ln(x)-4}{\left(1-ln^2(x)\right)}dx \overset{x=e^x}{\cong} \int_2^3 \frac{x-4}{(1-x^2)}dx = \frac{1}{2}\int_2^3 \frac{x-4}{x+1}dx - \frac{1}{2}\int_2^3 \frac{x-4}{x-1}dx =$$

$$\frac{1}{2}\int_2^3 \frac{x+1-5}{x+1}dx - \frac{1}{2}\int_2^3 \frac{x-1-3}{x-1}dx = \frac{1}{2}\int_2^3 dx - \frac{5}{2}\int_2^3 \frac{dx}{x+1} - \frac{1}{2}\int_2^3 dx + \frac{3}{2}\int_2^3 \frac{1}{x-1}dx =$$

$$-\frac{5}{2}\int_2^3 \frac{dx}{x+1} + \frac{3}{2}\int_2^3 \frac{1}{x-1}dx = -\frac{5}{2}(\ln(x+1))\Big|_2^3 + \frac{3}{2}(\ln(x-1))\Big|_2^3 =$$

$$-\frac{5}{2}(\ln(4)-\ln(3)) + \frac{3}{2}(\ln(2)-\ln(1)) = -\frac{5}{2}\ln(4) + \frac{5}{2}\ln(3) + \frac{3}{2}\ln(2) =$$

$$-5\ln(2) + \frac{3}{2}\ln(2) + \frac{5}{2}\ln(3) = \frac{5}{2}\ln(3) - \frac{7}{2}\ln(2)$$

$$\int_{e^2}^{e^3} \frac{\ln(x)-4}{\left(1-ln^2(x)\right)}dx = \frac{5}{2}\ln(3) - \frac{7}{2}\ln(2)$$