ROMANIAN MATHEMATICAL MAGAZINE

**Find a closed form:**

$$\int_0^{\ln(2)} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1}dx$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Mirsadix Muzefferov-Azerbaijan*

$$\int_0^{\ln(2)} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1}dx \overset{e^x\to t}{\cong} \int_1^2 \frac{2t^3+t^2-1}{t(t^3+t^2-t+1)}dt$$

$$\frac{2t^3+t^2-1}{t(t^3+t^2-t+1)} = \frac{A}{t} + \frac{Bt^2+Ct+D}{t^3+t^2-t+1}$$

$$\begin{cases} A+B=2 \\ D-A=0 \\ A+C=1 \\ A=-1 \end{cases} -->> \begin{cases} A=-1 \\ B=3 \\ C=2 \\ D=-1 \end{cases}$$

$$\frac{2t^3+t^2-1}{t(t^3+t^2-t+1)} = -\frac{1}{t} + \frac{3t^2+2t-1}{t^3+t^2-t+1}$$

$$\int_1^2 \frac{2t^3+t^2-1}{t(t^3+t^2-t+1)}dt = -\int_1^2 \frac{1}{t}dt + \int_1^2 \frac{3t^2+2t-1}{t^3+t^2-t+1}dt =$$

$$-\ln(t)\Big|_1^2 + \int_1^2 \frac{d(t^3+t^2-t+1)}{t^3+t^2-t+1} = -\ln(2) + \ln\left(t^3+t^2-t+1\right)\Big|_1^2 =$$

$$-\ln(2)+\ln(11)-\ln(2) = \ln\left(\frac{11}{4}\right)$$

$$\int_0^{\ln(2)} \frac{2e^{3x}+e^{2x}-1}{e^{3x}+e^{2x}-e^x+1}dx = \ln\left(\frac{11}{4}\right)$$