

Find:

$$\Omega = \int_2^4 \frac{{}^{2024}\sqrt{\ln(9-x)}}{{}^{2024}\sqrt{\ln(9-x)} + {}^{2024}\sqrt{\ln(x+3)}} dx$$

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Solution by Tapas Das-India

$$\begin{aligned} y = 6 - x &\Rightarrow x = 6 - y \Rightarrow dx = -dy \\ x = 2 &\Rightarrow y = 4, \quad x = 4 \Rightarrow y = 2 \end{aligned}$$

$$\Omega = \int_4^2 \frac{{}^{2024}\sqrt{\ln(9-6+y)}}{{}^{2024}\sqrt{\ln(9-6+y)} + {}^{2024}\sqrt{\ln(6-y+3)}} (-dy)$$

$$\Omega = \int_2^4 \frac{{}^{2024}\sqrt{\ln(3+y)}}{{}^{2024}\sqrt{\ln(3+y)} + {}^{2024}\sqrt{\ln(9-y)}} dy$$

$$\Omega = \int_2^4 \frac{{}^{2024}\sqrt{\ln(3+x)}}{{}^{2024}\sqrt{\ln(3+x)} + {}^{2024}\sqrt{\ln(9-x)}} dx$$

$$2\Omega = \int_2^4 \frac{{}^{2024}\sqrt{\ln(9-x)} + {}^{2024}\sqrt{\ln(x+3)}}{{}^{2024}\sqrt{\ln(9-x)} + {}^{2024}\sqrt{\ln(x+3)}} dx$$

$$2\Omega = 4 - 2 \Leftrightarrow \Omega = 1$$