



Proposed by Sonu Aarnav-India

Solution by Daniel Sitaru-Romania

$$\begin{aligned}
 \text{Green area} &= \int_0^1 (g(x) - f(x)) dx = \int_0^1 \left(x \tan^{-1}\left(\frac{1}{x}\right) - x \tan^{-1}x \right) dx = \\
 &= \int_0^1 \left(x \left(\frac{\pi}{2} - \tan^{-1}x \right) - x \tan^{-1}x \right) dx = \int_0^1 \left(\frac{\pi x}{2} - 2x \tan^{-1}x \right) dx = \\
 &= \frac{\pi}{2} \left(\frac{1}{2} - \frac{0}{2} \right) - 2 \int_0^1 \left(\frac{x^2}{2} \right)' \tan^{-1}x dx = \frac{\pi}{4} - 2 \left(\frac{1}{2} \right) \tan^{-1}1 + 2 \int_0^1 \frac{x^2}{2} \cdot \frac{1}{x^2 + 1} dx = \\
 &= \frac{\pi}{4} - \frac{\pi}{4} + \int_0^1 \frac{x^2 + 1 - 1}{x^2 + 1} dx = \int_0^1 \frac{x^2 + 1}{x^2 + 1} dx - \int_0^1 \frac{1}{x^2 + 1} dx = \\
 &= 1 - \tan^{-1}1 + \tan^{-1}0 = 1 - \frac{\pi}{4}
 \end{aligned}$$