# ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\Omega = \lim_{n\to\infty} \frac{1}{n^2}\sum_{k=0}^{n} \ln\binom{n}{k}$$

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*Solution by Pham Duc Nam-Vietnam*

$$L = \lim_{n\to\infty} \frac{1}{n^2}\sum_{k=0}^{n} \ln\left(C_n^k\right) = \lim_{n\to\infty} \frac{1}{n^2}\ln\left(\prod_{k=0}^{n} C_n^k\right)$$

$$* \lim_{n\to\infty} \frac{\ln\left(\prod_{k=0}^{n+1} C_{n+1}^k\right) - \ln\left(\prod_{k=0}^{n} C_n^k\right)}{(n+1)^2 - n^2} = \lim_{n\to\infty} \frac{\ln\left(\prod_{k=0}^{n} \frac{n+1}{n+k-1}\right)}{2n+1}$$

$$= \lim_{n\to\infty} \frac{\ln\left(\frac{(n+1)^{n+1}}{(n+1)!}\right)}{2n+1} = \lim_{n\to\infty} \frac{(n+1)\ln(n+1) - \ln\left((n+1)!\right)}{2n+1}$$

$$* \lim_{n\to\infty} \frac{(n+2)\ln(n+2) - \ln\left((n+2)!\right) - (n+1)\ln(n+1) + \ln\left((n+1)!\right)}{2n+3-2n-1}$$

$$= \frac{1}{2}\lim_{n\to\infty}\left(n\ln\left(\frac{n+2}{n+1}\right) + 2\ln\left(\frac{n+2}{n+1}\right)\right) = \frac{1}{2}\lim_{n\to\infty}\left(\ln\left(1+\frac{1}{n+1}\right)^{n+1\frac{n}{n+1}} + 2\ln\left(\frac{n+2}{n+1}\right)\right)$$

$$= \frac{1}{2}\lim_{n\to\infty}\left(\ln\left(e^{\frac{n}{n+1}}\right) + 2\ln\left(1+\frac{1}{n+1}\right)\right) = \frac{1}{2}$$

$$\Rightarrow L = \frac{1}{2} \text{ by Stolz} - \text{Cesaro theorem}$$