

Find:

$$\Omega = \lim_{x \rightarrow 0} \frac{\ln(2 - \cos^2 x)}{x(e^x - 1)}$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Daniel Sitaru-Romania*

$$\begin{aligned} \Omega &= \lim_{x \rightarrow 0} \frac{\ln(2 - \cos^2 x)}{x(e^x - 1)} = \lim_{x \rightarrow 0} \frac{\ln(2 - (1 - \sin^2 x))}{x(e^x - 1)} = \lim_{x \rightarrow 0} \frac{\ln(1 + \sin^2 x)}{x(e^x - 1)} = \\ &= \lim_{x \rightarrow 0} \frac{x}{e^x - 1} \cdot \frac{\ln(1 + \sin^2 x)}{x^2} = 1 \cdot \lim_{x \rightarrow 0} \frac{\ln(1 + \sin^2 x)}{x^2} = \\ &= \lim_{x \rightarrow 0} \frac{1}{2x} \cdot \frac{2\sin x \cos x}{1 + \sin^2 x} = \lim_{x \rightarrow 0} \frac{\cos x}{1 + \sin^2 x} \cdot \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1 + \sin^2 x} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} = \\ &= \frac{\cos 0}{1 + \sin^2 0} \cdot 1 = 1 \end{aligned}$$