ROMANIAN MATHEMATICAL MAGAZINE

If:

$$\sum_{m=0}^{\infty}\left((-1)^{\left[\frac{m+1}{5}\right]}+(-1)^{\left[\frac{m-1}{5}\right]}\right)x^m=2$$

then find the value of the expression $x^5-x^3-x^2-x$

$[*]$ *is the floor function*

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$$\Omega=\sum_{m=0}^{\infty}(-1)^{\left[\frac{m+1}{5}\right]}x^m+\sum_{m=0}^{\infty}(-1)^{\left[\frac{m-1}{5}\right]}x^m=\Omega_1+\Omega_2=2$$

$$0<x<1;\ \ k=\left[\frac{m+1}{5}\right], n=\left[\frac{m-1}{5}\right], m\epsilon N_0;\ \ \begin{matrix} m+1<5, k=0 \\ m+1>5, k\epsilon N \\ m<1, \quad k\epsilon Z^- \end{matrix}$$

$$\Omega_1=\sum_{m=0}^{\infty}(-1)^{\left[\frac{m+1}{5}\right]}x^m=1+x+x^2+x^3-x^4-x^5-x^6-x^7-x^8+x^9\dots$$

$$=\sum_{m=0}^{3}x^m-\sum_{m=4}^{8}x^m+\sum_{m=9}^{13}x^m-\cdots$$

$$\Omega_2=\sum_{m=0}^{\infty}(-1)^{\left[\frac{m-1}{5}\right]}x^m=-1+x+x^2+x^3+x^4+x^5-x^6-x^7-\cdots-x^{10}+x^{11}+\cdots$$

$$\Omega_1+\Omega_2=2,\qquad 2x+2x^2+2x^3-2x^6-2x^7-2x^8+\cdots=2$$

$$(x+x^2+x^3)-x^5(x+x^2+x^3)+x^{10}(x+x^2+x^3)\dots=1$$

$$(x+x^2+x^3)\left(1-x^5+x^{10}-x^{15}+x^{20}-\cdots\right)=1$$

$$(x+x^2+x^3)\sum_{m=0}^{\infty}(-1)^m x^{5m}=1;\frac{x+x^2+x^3}{1+x^5}=1$$

$$-x^5+x+x^2+x^3=1\ ;\ \ x^5-x^3-x^2-x=-1$$